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**Spatial description of concrete
characteristics.**

**Descripció espacial de
característiques del formigó.**

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TREBALL FINAL DE GRAU

Abstract

This work has the objective to simulate the variation of the compressive resistance strength of concrete along the height of a structural member. The concrete compressive strength is calculated with a formula, which has been previously analysed. To calculate the distribution, an exploratory and a statistical analysis of data are carried considering different parameters that can affect. When exploration is completed, a transformed linear regression model is proposed. After, an interval of confidence is considered for the linear regression using the bootstrap method .

The most influential variables on the distribution are the day test time and the type of concrete (whether it is self-compacting concrete or not). Others, like the type of test (whether the compressive strength test is destructive or not) affects on the results of the tests but not on the distribution.

The obtained model takes into account all the parameters to compute the compressive resistance of the concrete and the distribution along the height of the structural member. Furthermore, an application has been created in order to compute it easily.

Contents

Acknowledgements	2
1. Introduction and Objectives	3
2. Computation of compressive strength	3
2.1 Analysis of the formulas	4
2.2 Bolomey formula	7
3. Distribution of the strength along its height	8
3.1 Classification of the data	10
3.2 Analysis of the data	12
3.3 Model to predict the variation of the strength concrete at day 28 . . .	18
3.4 Setting and interval of confidence	23
4. Theoretical example	24
4.1 App created	24
4.2 Example	24
5. Conclusions	27
6. Further work	28
Bibliography	29

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1. Introduction and Objectives

The main objective of this work is to predict the compressive strength of a given structural member and how it will be distributed along its height. To do so, this work has 4 main sections. The first consists of finding a formula from different sources. Then, all the formulas are proved with all the data collected and it is checked whether it fits correctly or not using statistical tools. The second step is focused on how this compressive strength varies from bottom to top. Different thesis and investigations are taken into account in order to collect information about this phenomenon. As none of the works found provides any mathematical model, the third step attempts to calculate a formula which models this variation and give an interval of confidence. Finally, an example is presented in order to show how the model created is used.

2. Computation of compressive strength

In literature, there are a great amount of formulas used to compute the concrete compressive strength according to the quantity of materials used. Some of them have been proposed using their own data, which has been obtained in a programme of tests. In this project, a total of 10 formulas have been used after a research of previous works, such as thesis and regulations.

The inputs for each formula vary significantly. Sayed-Ahmed (2012), from the Ryerson University, takes as inputs time, water, cement, metakaolin, silica fume, sand, aggregates and superplasticizer . It considered a statistical model to predict the compressive strength depending on the matrix mixtures used. Based on the same idea, Ahmad and Alghamdi (2014) proposed a statistical approach to obtain an optimum proportioning of concrete mixtures. They took into consideration the most important factors affecting the compressive strength: the cement content, the cement and water ratio and the fine and total aggregate ratio. They developed a polynomial regression in terms of these 3 factors.

The origin of each formula also differs. Some authors have chosen certain parameters that affect the concrete strength and they have used the results of their own experimental set to create a linear model. Others start from existing formulas and develop them to obtain a better one, like Zain et al. Considering the Abrams Law, more variables are added to the initial model and, as these variables are interrelated to each other

and there is a multiple dependency, a logarithmic transformation is applied obtaining a multivariable power equation. See the table 1 to check some of these formulas.

2.1 Analysis of the formulas

Firstly, the data of this section has been obtained from the thesis and reports considered before and from the thesis of Vilanova, A. (2009) and the free data set of Yeh, I. (2009). This benchmark dataset is made up to 1054 samples with information like the quantity of water per cubic meter, aggregate per cubic meter and cement per cubic meter and the result of concrete compressive resistance. Programmes Excel Microsoft and R programme (R Core Team, 2014) are used in this analysis.

The rule used to compare all equations was the following:

1. The ratio between the real compressive resistance and the value predicted for the analyzed formula are computed.
2. Then, the average and the standard deviation are calculated.

The most reliable formula was the Bolomey formula which had an average value of 1.15 (which means that the formula obtains lower values than the laboratory results) and a standard deviation of 0.29. The figure 1 shows the histogram of the values obtained from the ratio.

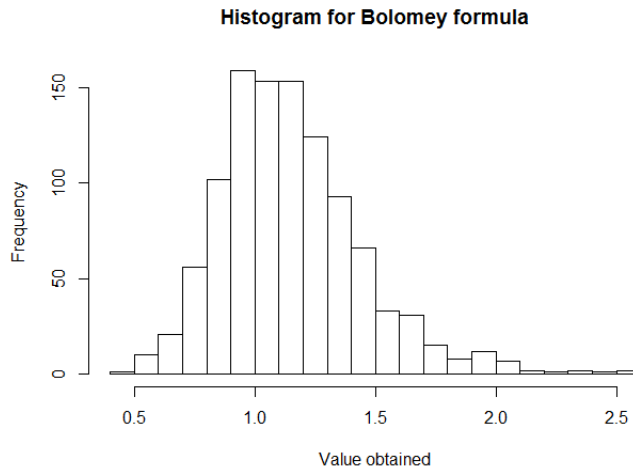


Figure 1: Histogram of the ratio values for the Bolomey formula.

Table 1: Summary of different formulas used to assess the compressive strength of concrete.

FORMULA	PARAMETERS	EXPLANATION	Reference
$f'_c = k_1 + k_2 \exp\left(\frac{-E_a}{k_2 T}\right) \text{ where}$ $E_a = \left(\frac{kw}{\sum_{i=1}^n a_i cm_i} + \sum_{i=1}^n b_i A_i\right)$	<p>T is the day taken, k, k_1 and k_2 are values fitted in a linear regression. E_a represents the matrix mixture value and uses the quantity of cement, water, fly ash, sand and silica fume.</p>	<p>This model considers a mechanical formula and the mixture matrix (E_a) as starting formulas and combines them using a linear regression with the data collected in the laboratory</p>	[13]
$f_t = A \log(t) + B$	<p>t is the compressive strength age in days and A and B are parameters calculated with the data provided by the laboratory.</p>	<p>Find a formula to fit the data obtained from the laboratory.</p>	[1]
$f_{28} = (a_0)(C)^{a1}(W)^{a2}(FA)^{a3}(CA)^{a4}(\rho)^{a5}\left(\frac{W}{C}\right)^{a6}$	<p>W is the water, C is the cement, FA represents the quantity of sand, CA is the aggregate and ρ is the density of concrete (approx. 2400 Kg per cubic meter). All values are given in Kg/m^3.</p>	<p>The objective of this research project is to find a non-linear model to calculate the compressive strength of concrete at ages of 7 and 28 days.</p>	[21]
$f'_c = -61.24 - 0.056Q_c - 19.87 \exp(2.083R_{w/cm}) + 183.45R_{FA/TA}^{0.119}$	<p>Q_c is the quantity of cement, $R_{w/cm}$ is the ratio of water and cement and $R_{FA/TA}$ is the ratio between fine an total aggregate. All the values are introduced in Kg/m^3.</p>	<p>The objective of this thesis focuses on finding a formula to calculate an optimum proportioning of concrete mixtures of 92% of ordinary Portland cement and 8% of silica fume.</p>	[3]

FORMULA	PARAMETERS	EXPLANATION	Reference
$A_0 \left(\frac{W}{CM}\right)^{A_1} \left(\frac{FA}{CM}\right)^{A_2} \left(\frac{CA}{CM}\right)^{A_3}$	<p>W represents the water content, CA the content of coarse aggregate and FA the fine aggregate. The values given to $A_i, i = 0, \dots, 6$ depend on the granulometric curve of the aggregate and the quantity of fly ash used.</p>	<p>The parametrization of this formula has also taken into account the type of granulometric curve and it has analyzed the values obtained of a previous laboratory experimental work.</p>	[6]
$f_{c28} = 1237.66 - 0.695(X_1) - 0.292(X_2) - 0.501(X_3) - 0.53(X_4) - 1.117(X_5) + 1.013(X_6) - 606.478(X_7) + 3.673(X_8) - 30.994(X_9) + 12.887(X_{10})$	<p>$X_i, i = 1, \dots, 10$ represents respectively the quantity of: cement, coarse aggregate, fine aggregate, slag, fly ash, chemical admixture, water to binder ratio, age in days, moisture content and rebound value.</p>	<p>This study estimates the strength of concrete in non-destructive tests. Then, regression analysis are calculated.</p>	[9]
$f'_c = \frac{A}{B^{w/c}}$	<p>A and B are constants related to the characteristics of granulometry and w and c are the water and the cement respectively.</p>	<p>This is the Abraham's formula and it is used in the thesis of Zain et al.</p>	[21]

2.2 Bolomey formula

The Bolomey formula is the formula (1),

$$f_{c28} = f_{cm28} * k_b * \left(\frac{C + kA}{W * 0.8 + \rho_W V} - 0.5 \right) \quad (1)$$

Where:

- f_{c28} is the compression of the concrete in 28 days in MPa.
- f_{cm28} is the resistance of the cement paste at the age of 28 days in MPa. If the type of the cement is known, we add 12.5 MPa to the type resistance, but if the type is unknown, a resistance of 45MPa is considered.
- k_b is a constant related to mechanical properties of the granulate and the quality of the adherence between paste and granulate. It was taken as a constant value and equal to 0.5.
- C is the quantity of cement in Kg/m³.
- kA is the quantity of addition multiplied by a constant.
- W is the water used in 1 m³ of concrete and it is expressed in Kg/m³. It is multiplied by 0.8 because the formula takes the effective water, not the total water.
- ρ_W is the density of water in Kg/l.
- V is the content of air in the concrete in l/m³. Most of the data did not have that piece of information, then it has been taken equal to 15 l/m³.

3. Distribution of the strength along its height

Normally, the distribution of the compressive resistance is considered uniform in a structural concrete member. However, in some of the researched literature the variation along the height has been analyzed. The article written by Suprenant, B. A. (1994) published by the National Ready Mixed Concrete Association (NRMCA) focuses on that phenomenon. Firstly, it gives the difference between cylinders and cores and how cores are used to analyze the variation along the height.

In that work, cylinders are referred to the samples with a cylindrical shape created in a laboratory with the delivered concrete and they are tested in an uni-axial compression test when they have a certain age. But cores are cylindrical samples which are obtained by drilling from the *in situ* concrete. They are also tested in an uni-axial compression test.



Figure 2: Concrete compression test. Reference: Asian Institute of Technology. Thailand

As the Suprenant's thesis explains, the cylinder strength represents the quality of the concrete delivered. In this strength, the variables of concrete batching, mixing, transportation and curing influence the final compressive resistance. However, in core tests, the variables of the quality of placement, distance of the core from bottom and consolidation also affect. The drilling direction is supposed not to influence on the results.

Some of the analyzed thesis such as *In-Situ Mechanical Properties of Wall Elements Cast Using Self-Consolidating Concrete* (Khayat, K. H. et al.) followed the normative ACI 318, which specifies the necessity of a cure test if any of the strength tests of laboratory-cured cylinders falls below the specified f_c by more than 500 psi (3.45 MPa) or

if tests of field-cured cylinders indicate deficiencies in protection and curing. Regardless of the ACI normative, all the reports used cores to know the strength resistance along the height of the member except the papers of Ranjbar, M. M. et al. (2011) in which a Ultrasonic Pulse Velocity was used.

All the sources indicate 2 apparent causes for the strength variation:

- Greater static pressure caused by the concrete above increases the strength at the bottom.
- Due to bleed of water, there is a higher water-cement ratio and a decrease in strength at the top.

The papers and thesis of the consulted literature have provided data for the present work. Despite the fact that none of them had a mathematical model to estimate the variations, graphic approaches were generated. Figures 3 and 4 belong Suprenant's thesis and they attempt to give a graphical interpretation of the compressive resistance's variation, and figure 5 gives a more accurate approximation, although it does not present any mathematical formula. A new model has been developed in order to compute this variation.

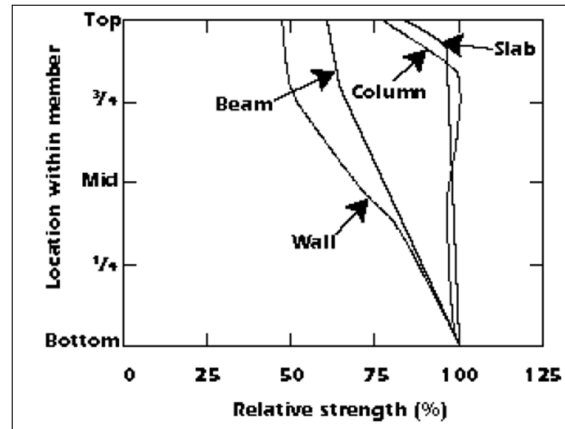


Figure 3: Approximation considering the relative position of the core for different structural members. Reference: [16]

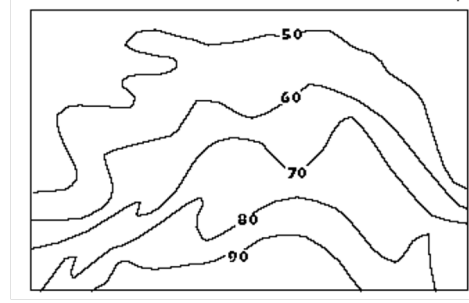


Figure 4: Predicted variation of compressive resistance for a wall. Neither units nor lengths are given. Reference: [16]

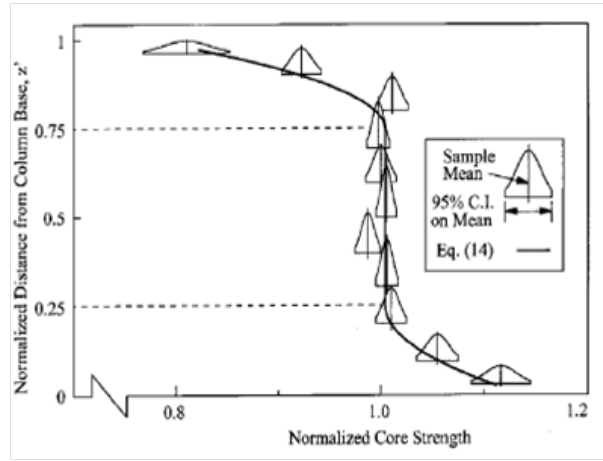


Figure 5: Predicted variation of compressive resistance for any structural member using the relative position of the core. Reference:[10]

3.1 Classification of the data

The table 2 shows the parameters that have been considered in the study.

Other variables like how the concrete was placed and curing procedures have been neglected because there was not enough information in all the sources.

Table 2: Parameters used to classify the data.

PARAMETER	EXPLANATION
Compressive strength of the core	It gives the compressive strength value of the core in MPa
Position of the core	The distance from bottom of the core in the structural member (in cm).
Total height of the structural member	The total height of the structure.
Concrete strength of cylinder test	The compressive resistance strength value obtained in an uni-axial test made to a concrete cylinder.
Relative height of the core	The ratio between the position of the core and the total height of the structural member. That value takes numbers from 0 to 1.
Relative strength of the core	The ratio between the compressive strength of the core and the compressive strength of the cylinder test. It is an adimensional value which takes numbers from 0 to 1.2 .
Wall or column variable	It classifies the type of the structure which every core comes from. It can be a wall or a column.
Steel bar variable	It classifies whether the structure which every core comes from has steel bars.
Destructive test variable	It classifies whether the compressive strength of a core has been made with a destructive or non-destructive test.
SCC variable	It classifies whether the concrete used was a self-compacting concrete or not.
Day test variable	The day when tests were made to cores. There are the values of 18, 19, 28 and 91 days.

3.2 Analysis of the data

The dataset obtained from the literature has been unified and formatted. It contains 278 observations with 11 variables. R programme (R Core Team, 2014) has been used to analyze. The figure 6 shows all the data taking the height of the point and resistance of compression as variables.

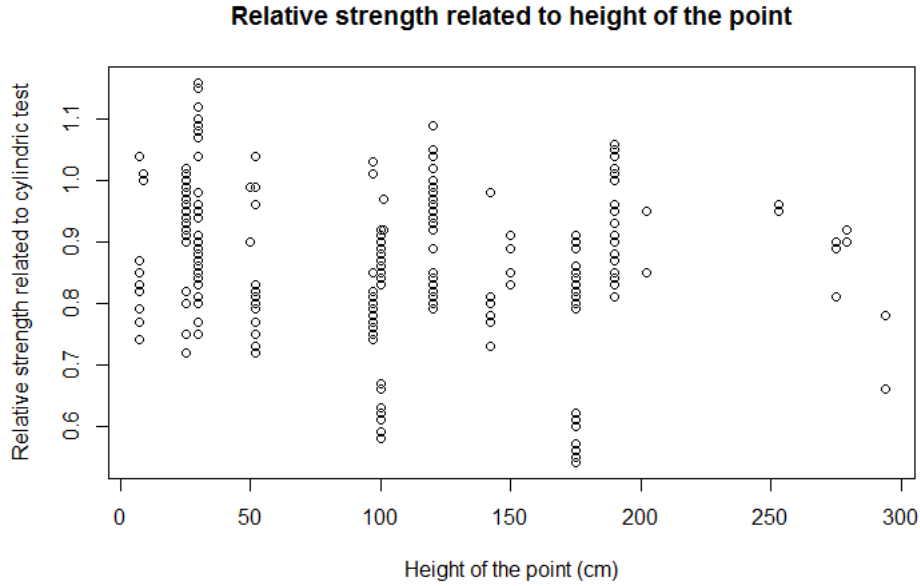


Figure 6: Scatterplot of the relation between the height of the point and the relative resistance obtained.

The goal is to predict the relative strength as a combination of other predictors. Some of them are continuous and others discrete. An exploration of the data is firstly made.

- The existence of steel bars, test type of the cores, the day of the test, the type of structure and the SCC category are categoric variables.
- The relative strength, the height of the core, the height of the structure and the strength of the cylinder test are continuous variables.

As most of the variables are categoric, a boxplot and a variance analysis are carried out to determine analytically the importance of the variable on the results.

The *column or wall* variable is taken as an example. Firstly, scatterplot (figure 7) is made in order to have a first overview about the influence of that variable on the distribution.

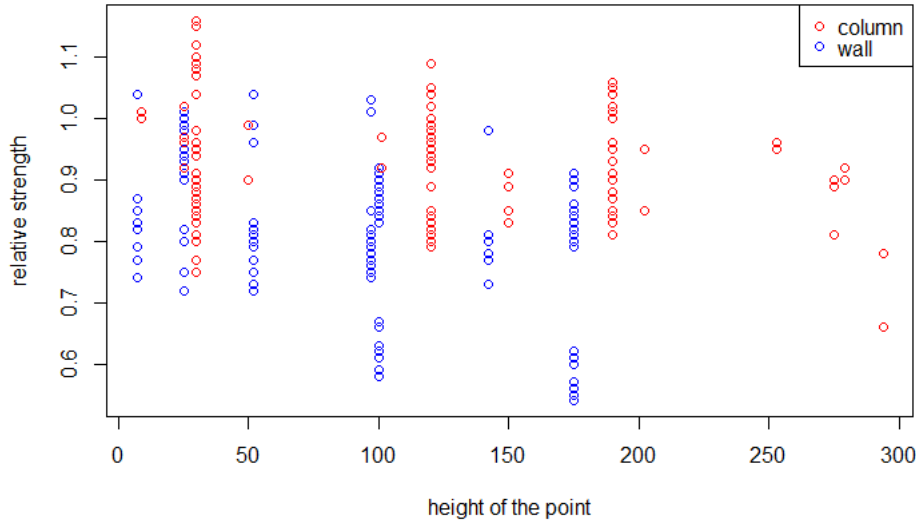


Figure 7: Scatterplot which classifies the data if they come from a wall or a column structure.

As it can be seen in scatterplot from figure 7, it seems that there is a certain influence on how the relative strength varies along the height depending on the type of structure.

A boxplot is also made and it shows a certain influence on the type of the variable. Finally, a contrast of hypothesis is performed with a level of statistical significance equal to 0.01. The equality of variances is analyzed with a Bartlett test.

$$H_0 : \sigma_{\text{wall group}}^2 = \sigma_{\text{column group}}^2$$

$$H_1 : \sigma_{\text{wall group}}^2 \neq \sigma_{\text{column group}}^2$$

As a p-value of 0.15 is obtained, the equality of variances cannot be rejected. After, the equality of averages is analyzed with a linear regression which shows that the *wall*

or *column* variable has an influence on the results.

The same process has been performed for all the variables. However, the variable that has the strongest influence on results is the *day test* variable (the day when tests were carried). As it had been expected, the later the tests are made, the higher the relative resistance to compression is. Boxplot in figure 8 shows the difference in relative resistance for different days of test.

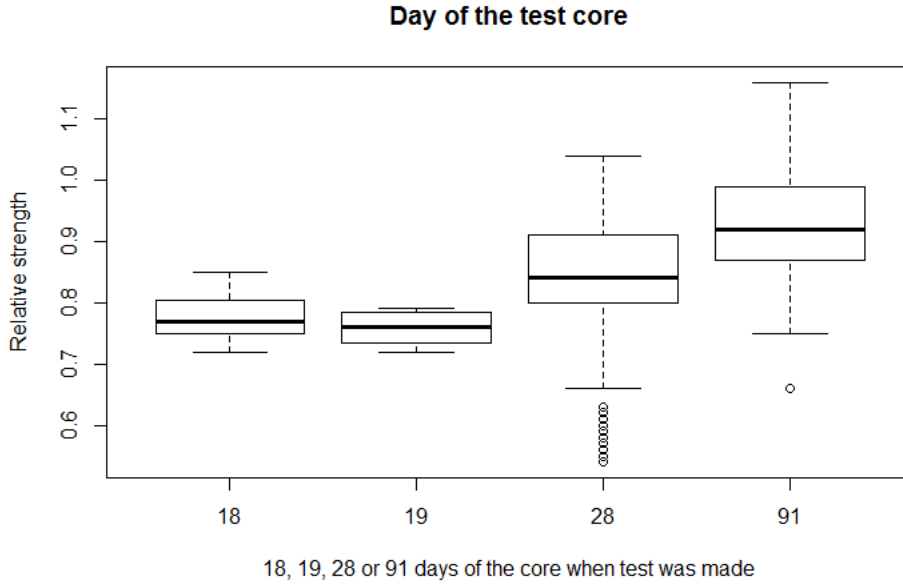


Figure 8: Boxplot considering the influence of day test.

As the *day test* variable has a strong influence on results, a group with only values of 28 day test data is selected from the test dataset. It contains 141 core tests. The *steel bar* and *wall or column* variables are not considered because all the samples belong to wall structures with steel bars. Graphic in figure 9 shows the influence of *SCC* variable.

Making a box-plot and an analysis of variances, a new group is created taking into account the *SCC* component. Then, in the *not SCC* and *28 day test* group of data, when it is plotted considering *destructive test* variable (figure 10), it shows that there are 2 trends where the *non-destructive test* values have lower results.

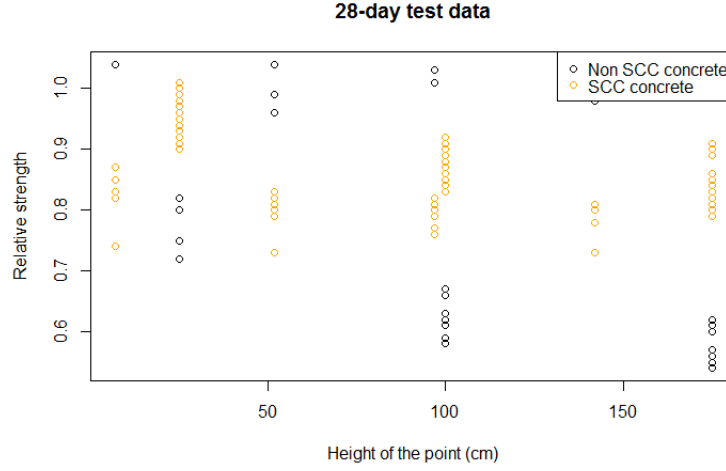


Figure 9: Scatterplot considering the influence of *SCC* variable.

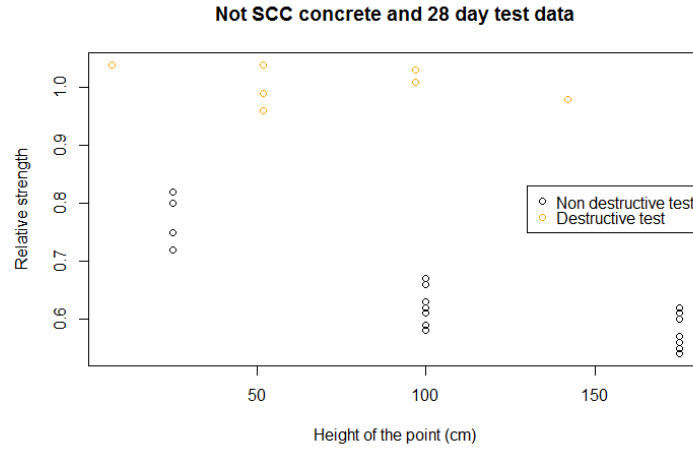


Figure 10: Scatterplot considering the influence of *destructive test* variable in not SCC concrete and 28 day test.

To analyze the influence of *destructive* and *not destructive* variables in this group, a Bartlett test is first made in order to know whether the variances are equal or not. As the variances are different, a Kruskal-Wallis test is made in order to confirm that the averages of these 2 groups are different (boxplot of the figure 11).



Figure 11: Boxplot considering the influence of *destructive test* variable.

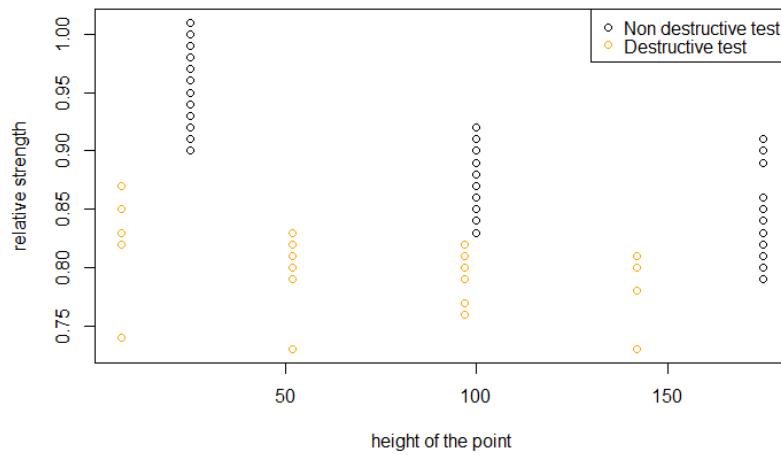


Figure 12: Scatterplot considering the influence of *destructive test* variable in the SCC concrete and 28 day test data.

After considering different analysis and making groups with the same trend, 4 groups of data are obtained (see figure 13).

- Group 1: Non destructive tests with not self-compacting concrete (28 day test). 21 observations.
- Group 2: Destructive tests with not self-compacting concrete (28 day test). 8 observations.
- Group 3: Non destructive tests with self-compacting concrete (28 day test). 84 observations.
- Group 4: Destructive tests with self-compacting concrete (28 day test). 28 observations.

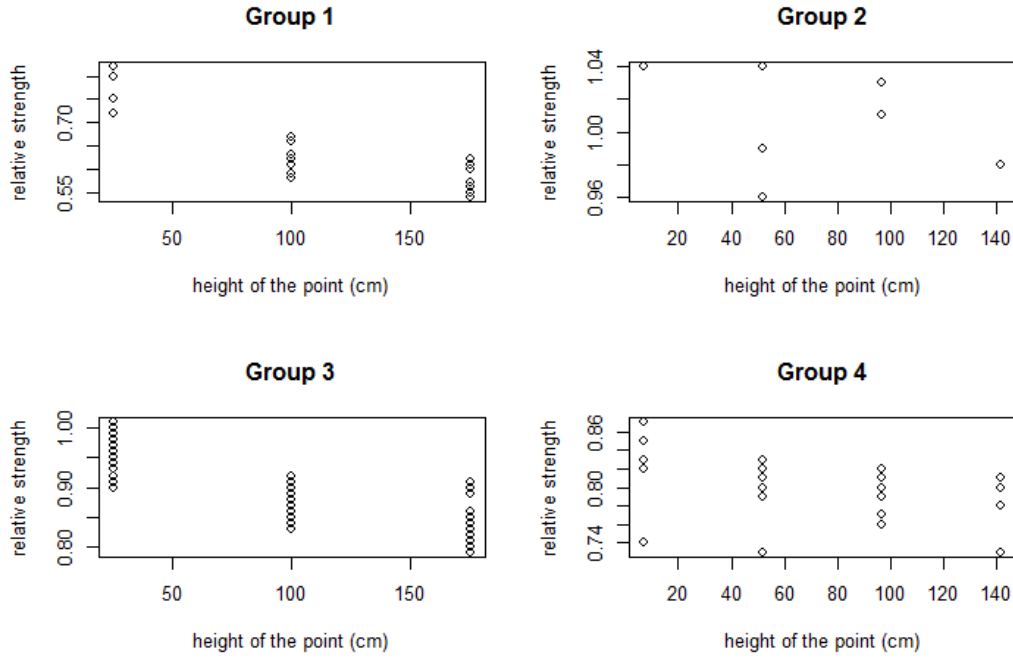


Figure 13: Graphics showing the general trend of the different groups.

3.3 Model to predict the variation of the strength concrete at day 28

As presented in section 3.2, there are 4 groups of data. The R program is used to analyze the data. Firstly, for each group, a linear model is used (equation (2)).

$$y = \beta_0 + \beta_1 x \quad (2)$$

where y is the relative compression resistance and x is the height of the analyzed point. Logarithms are in base e .

When the linear regression is launched, different analysis are made.

1. Residual hypothesis: Check whether the residuals are independent, with the same variance and follow a normal distribution.
2. Contrast F.
3. Check if any of the variables could be 0.
4. Quality of the model with an ANOVA table.

In order to consider their suitable scale, the 2 variables are log-transformed (formulas (3) and (4)).

$$Y = \text{logit}(y) = \log \left(\frac{y}{1.2 - y} \right) , \quad (3)$$

$$X = \log(x) , \quad (4)$$

A linear regression for group 1 is first performed. Graphics 14 show the residual analysis and table 3 gives the values of different parameters of the regression. The values for the coefficients are in table 7.

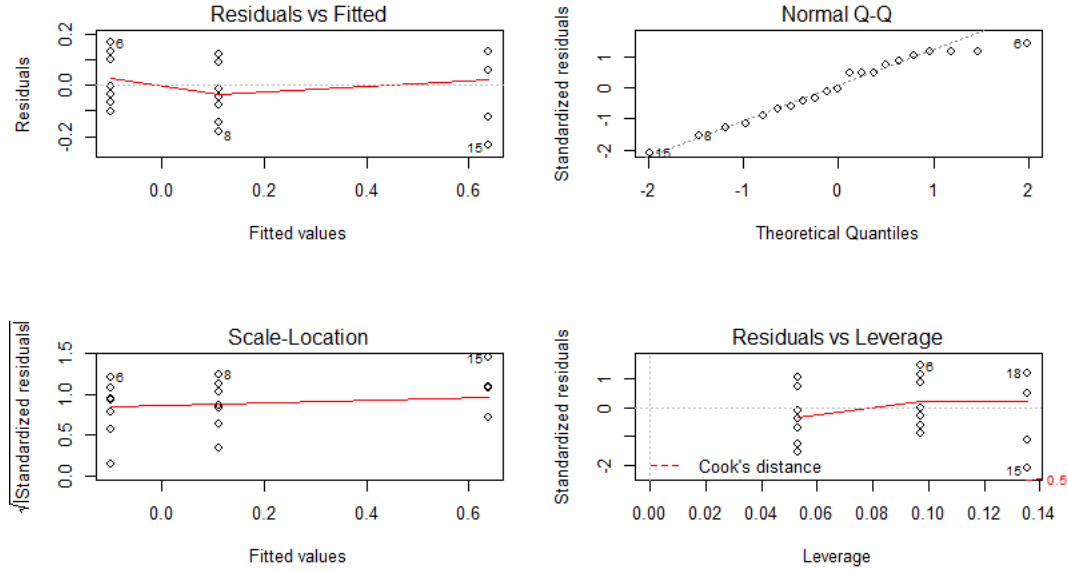


Figure 14: Graphics with the residual information about the linear regression.

Table 3: Summary of the linear regression of the 1st group.

Residual standard error: 0.1191 on 19 degrees of freedom Multiple R-squared: 0.8815, Adjusted R-squared: 0.8753 F-statistic: 141.4 on 1 and 19 DF, p-value: 3.027e-10

Group 2 is analyzed following the same procedure. Graphic 15 shows the residual fit. Its low quantity of observations is a problem to analyze properly this group. Furthermore, table 4 shows a high p-value and a poor R-square. In that case, the coefficients obtained for that group (see table 7) are not considered.

Table 4: Summary of the linear regression of the 2nd group.

Residual standard error: 0.1875 on 6 degrees of freedom Multiple R-squared: 0.1239, Adjusted R-squared: -0.0221 F-statistic: 0.8487 on 1 and 6 DF, p-value: 0.3925
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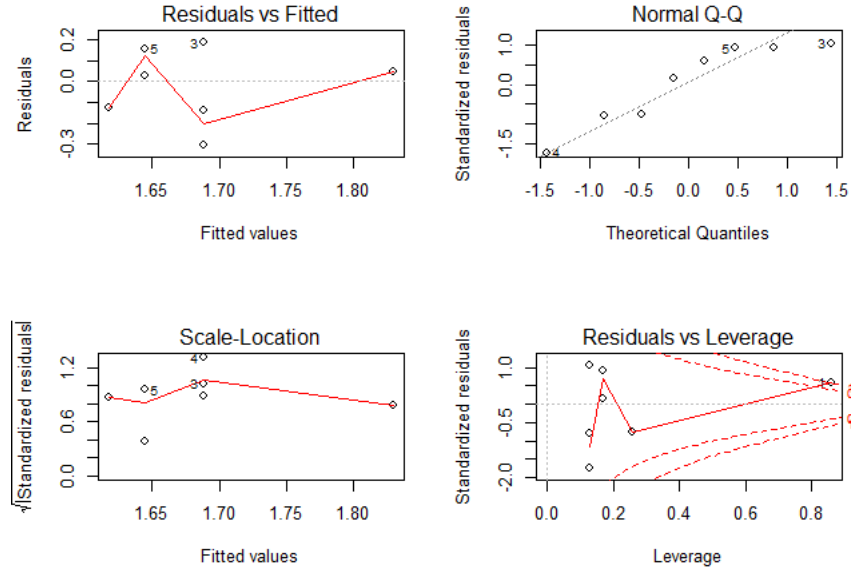


Figure 15: Graphics with the residual information about the linear regression of the 2nd group.

Group 3 is analyzed. Figure 16 shows different parameters of the residuals. According to *Normal Q-Q plot*, the residuals follow a normal distribution and the graphic *Residuals vs Leverage* show that none of the values has a strong effect on the linear regression. Table 5 gives the values of different parameters of the regression. The values for the coefficients are in table 7.

Table 5: Summary of the linear regression of the 3rd group.

Residual standard error: 0.1358 on 82 degrees of freedom Multiple R-squared: 0.7433, Adjusted R-squared: 0.7402 F-statistic: 237.5 on 1 and 82 DF, p-value: 2.2e-16

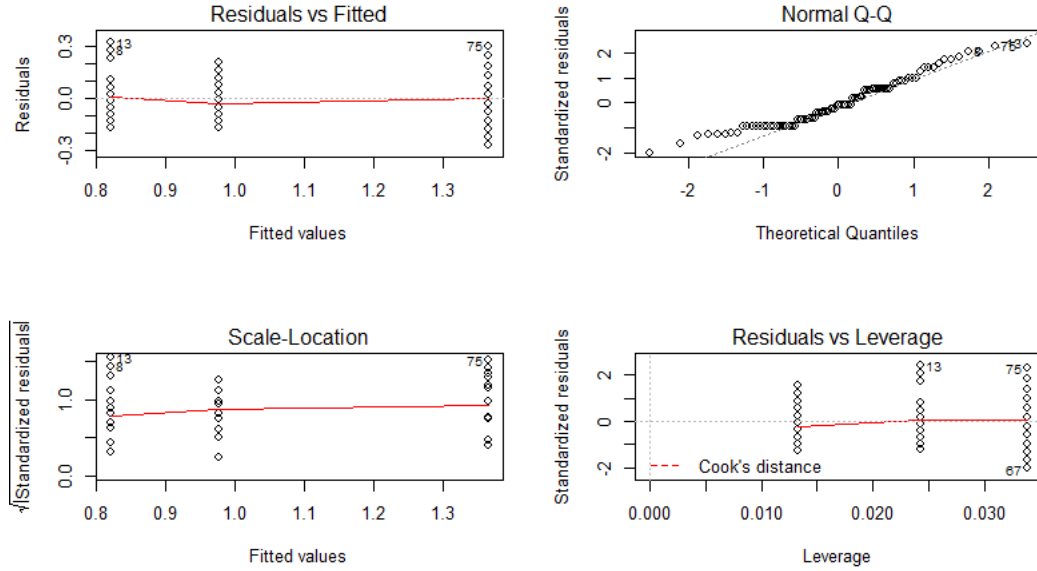


Figure 16: Graphics with the residual information about the linear regression of 3rd group.

Table 6: Summary of the linear regression of the 4th group.

Residual standard error: 0.06366 on 22 degrees of freedom Multiple R-squared: 0.4906, Adjusted R-squared: 0.4675 F-statistic: 21.19 on 1 and 22 DF, p-value: 0.0001383
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For the group 4, there are points which are removed because they do not fit with the general trend. The summary is expressed in table 6. Figure 17 shows the residual graphics of the regression. The values for the coefficients are in table 7.

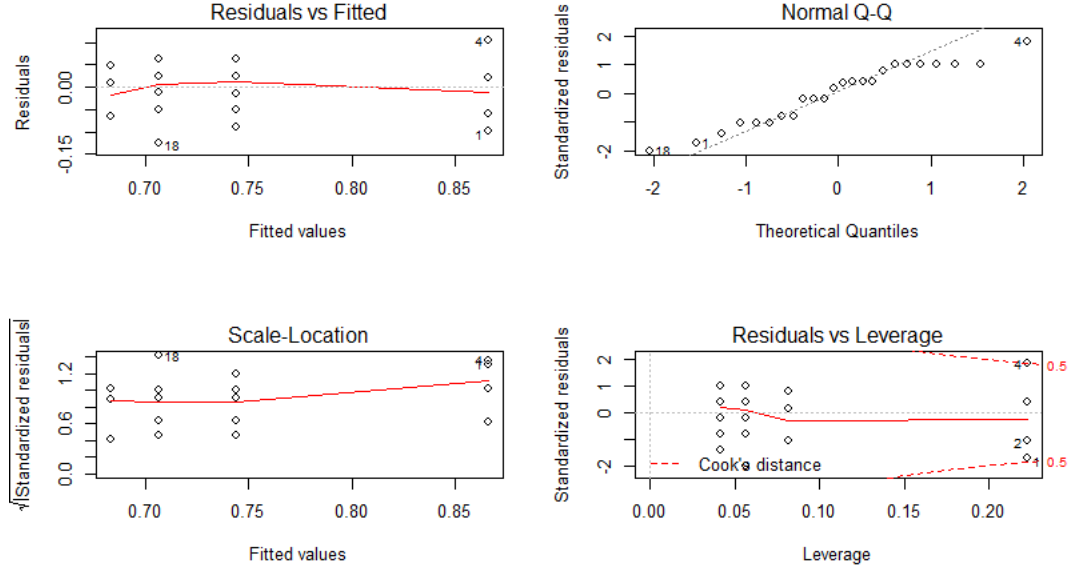


Figure 17: Graphics with the residual information about the linear regression of 4th group.

The results of this linear regression are very poor and this group is no more considered for further analysis.

Table 7: Summary of the parameters estimated in each regression.

Group	β_0	β_1
1	1.85	-0.37
2	1.96	-0.07
3	2.26	-0.27
4	0.98	-0.06

Groups 1 and 3 have the best approaches. Then, as they belong to 2 types of different concretes (not SCC and SCC respectively), 2 different formulas are proposed.

The formula used for not self-compacting concrete is the equation (5).

$$\log \left(\frac{y}{1.2 - y} \right) = 1.85 - 0.37 \log(x) , \quad (5)$$

and the formula used for self-compacting concrete is equation (6).

$$\log\left(\frac{y}{1.2-y}\right) = 2.26 - 0.27 \log(x) . \quad (6)$$

3.4 Setting an interval of confidence

In order to visualize the variability of the prediction, an interval of confidence for the linear regressions is set . To set this interval, the 2 linear regressions obtained in the previous chapter are bootstrapped using the R programme. As the article of *Bootstrapping Regression Models in R* (Fox, J. and Weisberg, S., 2012) explains, bootstrap is a general approach to statistical inference based on building a sampling distribution for a statistic by resampling from the data at hand. In this work, the type of bootstrap used has been the random-x or case resampling, which refits the predictor values of the linear regression depending on the level of confidence chosen. 2 levels of confidence has been taken into account: one of 5% and 95% and the other 25% and 75%.

For the linear regressions of groups 1 and 3, this method has given the intervals of confidence shown in tables 8 and 9.

Table 8: Different values for the predictors in the regression for group 1.

Predictor	5%	25%	75%	95%
β_0	1.60	1.75	1.96	2.11
β_1	-0.433	-0.40	-0.35	-0.32

Table 9: Different values for the predictors in the regression for group 3.

Predictor	5%	25%	75%	95%
β_0	2.12	2.20	2.32	2.40
β_1	-0.31	-0.29	-0.26	-0.24

4. Theoretical example

4.1 App created

Using the Shiny package of R programme (R Core Team, 2014), an app has been created. The app allows to include the parameters of the model and calculate the distribution and the expected resistance. Note that y-axis gives the compressive resistance in MPa which is the multiplication of the predicted resistance for concrete by the relative resistance distribution. After introducing all the paramaters, this tool generates 3 lines: the area contained into the lines blue and red represent the interval of confidence, and the green represents the predicted values for the linear regression.

4.2 Example

To show how this prediction model works, an example is showed.

A concrete beam of 2m tall is built. Consider self-compacting concrete with properties shown in table 10 .

Table 10: Characteristics of concrete used.

Cement type	Quantity of cement (Kg/m ³)	Quantity of water (Kg/m ³)
Ordinary Portland cement. 32.5 R	320	200

Which is the compressive resistance of a point located at 1m from the bottom? Consider an interval of 5% and 95%.

Considering the chosen formula to calculate the compressive resistance, a value of 29 MPa is obtained. Note that the quantity of trapped air has been considered as 15 l/m³ and the resistance of cement equal to $32.5 + 12.5 = 45$ MPa. Other parameters have been taken by default. Launching now the app to calculate the expected distribution, a point located at 1m from the bottom will have a compressive resistance between 24 and 28 MPa.

Prediction of Compressive Strength along the height of a structural member. 28 day concrete.

Total height of structural member. Between 100 and 300 cm (cm):

200

Position of the core studied (cm):

100

Choose the type of concrete:

Self-compacting concrete (SCC) ▼

Quantity of cement (Kg/m3):

320

Resistance of cement in 28 days (MPa). If unknown, 45 MPa:

45

Quantity of mineral admixtures (Calcareous or siliceous fly ash, silica fume, blast furnace slag and metakaolin) (Kg/m3):

0

Parameter for mineral admixture:

0.3

Quantity of water (Kg/m3):

200

Parameter Kb (constant related to mechanical properties of the granulate and the quality of the adherence between paste and granulate). Between 0.45 and 0.65:

0.5

Quantity of trapped air (l/m3). If unknown, 15 l/m3:

15

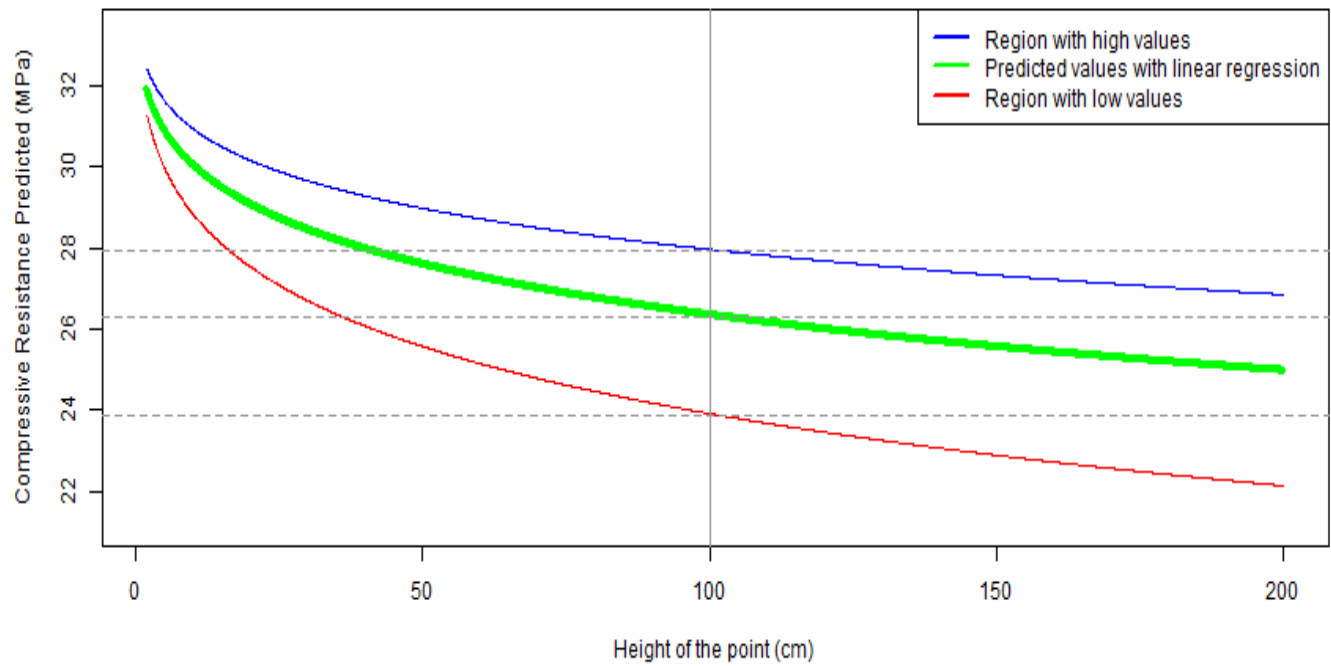
Choose the interval of confidence:

5% and 95%

▼

Predicted resistance for concrete (MPa): 29

Prediction of Compressive resistance along the height of structural member



5. Conclusions

This work gives a model to compute and analyze the variation of the compressive resistance in a structural member for 28 day *in-situ* concrete. The formula to calculate the compressive resistance according to the parameters of concrete has followed a severe procedure and it could be used in other academic works. Considering the variation of the resistance along the height, it is an approximation to understand the behaviour of a structural concrete member. Note that the model does not allow positions of the core studied of less than 2 cm because it tends to infinite close to 0.

The model proposed considers first the quantity of all materials used in order to calculate the compressive resistance. The variation of this resistance along the height is computed following the defined linear model set in section 3.3 *Model to predict the variation of the strength concrete at day 28*. As this variation is sensitive to the variability of many factors, 2 more linear regressions define a region of expected values of the compressive resistance (section 3.4 *Setting an interval of confidence*). The accuracy of this expected region can be defined choosing the type of interval of confidence.

The main conclusions derived from this work are expressed below.

- Despite not being considered explicitly in all resistant verifications, there is a variation in the resistance compressive strength along the height of any structural member that occurs because of gravity effects.
- The type of the test (*destructive test* or not) must be considered in order to avoid errors when a large dataset is analysed. This variable does not affect on the trend but it affects on the relative resistance.
- The type of concrete (SCC or not) influences the trend of the variation, being the SCC concrete more uniform than not SCC concrete.
- Some thesis state that the variability of the resistance depends on the resistance of the concrete used. However, in this thesis it has not been found as important.

6. Further work

This research project has attempted to create a model to analyze the variation of the compressive strength in concrete and give an interval of confidence for possible values. However, the variation of this distribution over time has not been taken into account and could be analysed for future analysis.

Moreover, variations of other parameters like modulus of elasticity and tensile strength have not been studied and they are also important in the behaviour of the concrete structure.

A simulation with finite elements of the present work could be proposed to get deeper into the analysis of the strength variation in a 2-D and 3-D model.

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